$$\boxed{z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)+j\sin\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)\right]}$$

$k = 0, 1, 2 \ldots n-1$

$$\boxed{z^{p} = r^{p}\left[\cos\left(p\theta+2\pi k p\right)+j\sin\left(p\theta+2\pi k p\right)\right]}$$

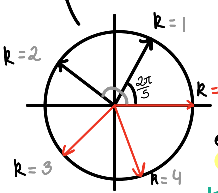
where $p = \frac{m}{n}$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{j\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)}$$

Conjugate - reflect in real axis

$-\pi < \theta \leq \pi$ aka. $\theta \in (-\pi, \pi]$

$= \arg z. = \tan^{-1}\left(\frac{Im}{Re}\right)$



Cheat: add $\frac{2\pi k}{n}$ to first root n times for each

each root equally spaced

remember $-\pi < \theta \leq \pi$ so in negative Im you must alter arg.

Solving power equations

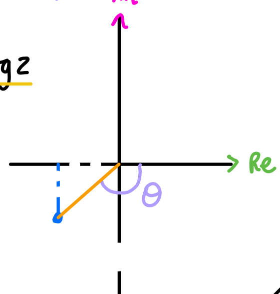
Argand Diagram

$x + jy \rightarrow r\cos\theta + j\, r\sin\theta$

$= r(\cos\theta + j\sin\theta)$

Polar form

argument (arg.)



$z + \bar{z} =$ real

$z - \bar{z} =$ imaginary

change sign for conjugate $\bar{z}$

$\ln z = \ln r + j(\theta + 2n\pi)$

$\ln z = \ln|z| + j\arg z$ (principal value)

$\ln z = \ln r + \ln e^{j\theta} \cdot 1$

$1 = e^{j2n\pi}$ Euler's

real

$j = \sqrt{-1}$

$z = x + jy$

Imaginary

$re^{j\theta}$ — Exponential form

$e^{j\theta} = \cos\theta + j\sin\theta$

Complex Numbers

- real terms grouped → $Re(z) = Re(\bar{z})$
- imaginary terms grouped → $-Im(z) = Im(\bar{z})$

Imaginary part does not include 'j'

If polynomial has only real coefficients

Roots

roots occur in conjugate pairs

e.g if cubic only has real coefficients

3 real roots or 2 conjugates and 1 real

Multiplying

expand as normal

remember $j^2 = -1$, $j^4 = 1$ etc.

$r_1 r_2\left[\cos(\theta_1+\theta_2)+j\sin(\theta_1+\theta_2)\right]$

easier in polar form

Dividing

Multiply top and bottom by conjugate of denomintator

$$\frac{z_1\bar{z}_2}{z_2\bar{z}_2}$$

'removed'

$\frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+j\sin(\theta_1-\theta_2)\right]$

Powers

$\left[r(\cos(n\theta)+j\sin(n\theta))\right]^n$

$r^n e^{jn\theta}$

to multiple angles

Multiple Angles to trig. polynomials

$\cos n\theta + j\sin n\theta = (c + js)^n$

1. expand with binomial theorem (sin and cos)
2. equate re parts for cos or im for sin
3. if you need one trig function, change with $s^2 + c^2 = 1$

e.g $s^2 = 1 - c^2$

Of trig. functions

$z^n = \cos n\theta + j\sin n\theta$

$$\therefore \boxed{\begin{aligned} z^n + z^{-n} &= 2\cos n\theta \\ z^n - z^{-n} &= 2j\sin n\theta \end{aligned}}$$

$\cos^5\theta$ in multiple angles:

$(2\cos\theta)^5 = \left(z + \frac{1}{z}\right)^5$

$32\cos^5\theta = z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$

group powers: $\left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$

$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$